TECHNICAL MEMORANDUM

No. K-10/61

REVIEW OF DATA ON INDUCED MASS

AND DRAG OF THE BASIC FINNER MISSILE

by

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Computation and Analysis Laboratory

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DAHLGREN, VIRGINIA

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Experimental data from Davidson Laboratory on induced mass and drag of the basic finner missile have been reevaluated at the Naval Weapons Laboratory. All but four of the runs must be discarded as determinations of induced mass because of uncertainty in velocity. It is concluded that an insight into the mechanism of induced mass can only be achieved through mathematical analysis. ( )

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ABSTRACT

Experimental data from Davidson Laboratory on induced mass and drag of the basic finner missile have been reevaluated at the Naval Weapons Laboratory. All but four of the runs must be discarded as determinations of induced mass because of uncertainty in velocity. It is concluded that an insight into the mechanism of induced mass can only be achieved through mathematical analysis.

FOREWORD

This report has been prepared in compliance with BUWEPS Directive RRRE 07 004/210-1/R009-02-003 dated 10 October 1960. Date of completion was 6 April 1961.

INTRODUCTION

An extensive effort has been made in various laboratories to determine the hydrodynamic characteristics of missiles. Quantitative data on forces and moments are available for steady flight. The interpretation and utilization of data for unsteady flight require a knowledge of the induced mass and moments of inertia of the entrained fluid. Theoretical studies of this induced mass have been limited to ideal fluids. No reliable information is available yet about how the induced mass varies with Reynolds number or acceleration modulus.

The California Institute of Technology has investigated the motion of the basic finner missile in vertical and horizontal flight. Quantitative interpretation of the horizontal runs probably will be possible after we know the dependence of induced mass on Reynolds number. Quantitative correlation of the vertical runs has not been possible without an assumption that the actual masses were incorrectly recorded. No reruns have been possible because the apparatus has been dismantled.

The Naval Weapons Laboratory proposed in reference 1 that programmed acceleration trials be run in a towing tank. The Davidson Laboratory has made such trials and has established new values for the steady state drag of the basic finner missile. The drag curve has a

mysterious hump, but no documentation of flow regimes is available to explain the cause of the hump. The instrumentation was barely adequate to measure the induced mass ($k_1 \sim 0.15$). The following sources of error are considered noteworthy.

a. The interior of the model was flooded with fluid. Although a portion of the interior was plugged, the pressure gradients during acceleration were those of a hollow shell filled with fluid. The equivalent mass of the fluid content was a substantial part of the internal mass, and this caused a loss of sensitivity.

b. The interior of the model was exposed to the fluid pressure at the rim of the flat base. The steady drag was correctly determined insofar as the base pressure is uniform. The effect of induced mass was probably in error because the pressure distribution from induced mass is not uniform over the base.

c. The velocity of the model could not be controlled to follow closely an ideal stepwise variation. The velocity records were recorded at too small a scale to be read with precision.

Although these errors largely obscure the induced mass, a reanalysis has been made at the Naval Weapons Laboratory in an attempt to retrieve some useful information.

TRAJECTORY INTEGRATIONS

The test conditions of the runs were forwarded by Davidson Laboratory in reference 2 and the original records were forwarded in reference 3. A catalog of the test conditions is given herewith in Table I and samples of the records are reproduced in Figures 1 and 2.

With the exception of the first four runs in Table I the velocities were recorded at much too small a scale. The smallest scale division on the speed records has been converted to speed increments through the use of a calibration chart which was supplied with reference 3. The speed increment per scale division is listed in

Table I. Even with the assumption that the speed records can be estimated correctly to one fifth of a scale division the uncertainty in speed is enough to mask the induced mass*. Thus there would be a 100% error in induced mass if there were an error of 0.36 (ft)/(sec) over a velocity range of 5 (ft)/(sec).

The uncertainty in the records from wiggles is clearly apparent from an inspection of Figures 1 and 2.

The original records from the Davidson Laboratory have been reanalyzed at the Naval Weapons Laboratory on the basis of a momentum-displacement correlation instead of a force-velocity correlation. The objective of the change in correlation was a reduction in the uncertainty from wiggles in the oscillographic records. The drag records were reevaluated to obtain a set of average drag forces whose summation would reproduce the area under the drag traces. The results of this reevaluation are documented in Appendix C.

Simplified trajectory integrations were performed on NORC with a uniform time interval of 0.5 (sec). The trajectory integrations utilized a mass coefficient m and a drag coefficient k. The mass coefficient m (slugs) for the test models is related to the added mass coefficient k_1 by the equation**

$$m = (1.01) + (0.50)\ k_1 \tag{1}$$

and the drag coefficient k (slugs)/(ft) is related to the drag coefficient C_D by the equation**

$$k = (0.0888)\ C_D \tag{2}$$

where C_D applies to the base area alone.

*In reference 1 the specification of accuracy was 0.5% of full scale.

**These equations are derived from a base area of 0.0916 $(ft)^2$, a total volume of 0.2599 $(ft)^3$, and a total weight of 32.48 (lb), as quoted in reference 4.

The accumulated error in impulse ε_n (lb) (sec) after n steps of integration is given by the equation

$$\varepsilon_n = \sum_{i=1}^{n} \left\{ \tfrac{1}{2} \bar{f}_i - (m_i\, v_i - m_{i-1}\, v_{i-1}) - \frac{(k_{i-1}\, v_{i-1}^2 + k_i\, v_i^2)}{4} \right\} \tag{3}$$

where $\bar{f}_i$ (lb) is the average force in the i^{th} interval and v_i (ft)/(sec) is the velocity at the end of the i^{th} interval. The coefficients were interpolated from a table of values at half intervals in velocity. A table of values is given in Appendix C. Thus the values m_i were computed from the equation

$$m_i = m_j + (2\, v_i - j)\ (m_{j+1} - m_j) \tag{4}$$

and the values k_i were computed from the equation

$$k_i = k_j + (2\, v_i - j)\ (k_{j+1} - k_j) \tag{5}$$

where j is the serial number in the table for that entry v_j which is next smaller than v_i. The results of computation were plotted on the CRT printer. A set of results is given in Appendix D where ε_n is plotted against v_n, and each point is labeled with the value of n. An error in m_j is reflected in the plots by a nonzero slope during increment of velocity and an error in k_j is reflected by a nonzero drift rate during stationary velocity. The entries in the table of coefficients are so adjusted by trial as to minimize the random deviation of the plots from the velocity axis.

Various adjustments of mass and drag were tested. The induced mass could be varied by 25% from the value which is reported in reference 4 without appreciable improvement in the error curves. The drag from the constant speed runs was not the optimum and an improvement in the error curves could be achieved through an adjustment of drag. Values of k_j from the constant speed runs are listed in the second column of Table II and the values of k_j after adjustment are listed in the third column of the table*. The adjusted values are basic to the error curves in Appendix D.

*Although constant values are listed in the table for low velocity and for high velocity these were never used in the actual integrations.

Even if all runs are rejected except the first four because of error in velocity, there is still a discrepancy in induced mass between runs 2 and 20.

FLOW ANALYSIS

Experimental determinations of induced mass heretofore have not been quantitative. It seems obvious that mathematical analysis is necessary for an insight into the characteristics of induced mass.

If a missile were accelerated suddenly from one constant velocity to another then a potential flow would be superimposed upon the preexisting flow. Since the potential flow would not satisfy the boundary condition of constant velocity at the surface of the missile, the potential flow would be modified gradually through a diffusion of vorticity. Meanwhile the drag would decay from a large initial value to a steady final value. The acceleration thus would initiate a greater total impulse than that required to create the potential flow.

If a rapid acceleration cycle were applied to the missile the diffusion of vorticity would not have time to develop and the induced mass would be just that of the classical potential flow. A computing program for potential flow over missiles has been developed by the Douglas Aircraft Company. Details are given in reference 5 and subsequent reports. The computing program can be applied to the basic finner missile.

If a slow acceleration cycle were applied to the missile the diffusion of vorticity would have time to develop and the total impulse would be the integrated result of differential increments of flow configuration.

A theoretical study of induced mass is underway at the Naval Weapons Laboratory. An initial model consisted of a pair of line vortices behind a cylinder. The effect of the vortices was found to be a decrease of induced mass. An acceleration moves the vortices closer to the cylinder, and diminishes the pocket of entrained fluid.

Although circulatory motion can be observed in the trailing wakes of cylinders, the concept that vorticity is concentrated in the pocket is illusory. Valid solutions of the Navier-Stokes equation show that the vorticity trails off from the cylinder in a vortex sheet from each separation point. There is relatively little vorticity in the pocket of entrained fluid, while there is even less potential gradient.

A new computing program for solving the Navier-Stokes equation is now in preparation. The new program will give time dependent solutions for flow past a cylinder. Line vortices are placed at the intersections in a grid. The rate of change of vortex strength at each grid point is determined by finite difference approximations of the diffusion and convection of vorticity. Stream function is determined by the summation of contributions from each line vortex. Storage requirements in the calculator are minimized by the use of a polar grid. This program will provide the first determination of a variation of induced mass with Reynolds number.

More information about flow regimes is needed. Possibly small models of the basic finner missile could be moved through a bentonite suspension in a tank with polaroid windows. Photographs of the double refraction would show the onset of turbulence at various points on the missile.

RECOMMENDATIONS

1. It is recommended that further tests on induced mass be sponsored at Davidson Laboratory, but only if all of the following specifications are met:

a. The power drive and recording system be modified to give a better control and a more precise determination of the velocity.

b. The model be mounted on side struts instead of the base sting (as recommended by Davidson Laboratory).

c. The interior of the model be sealed off from fluid contact (with dynamometer in struts).

2. It is recommended that a project be established at a hydraulic laboratory for the photography of flow regimes.

3. It is recommended that a project be established at Douglas Aircraft Company for the computation of potential flow over the basic finner missile.

4. It is recommended that the programming and calculation of the vortex strength behind a cylinder be continued at the Naval Weapons Laboratory to the point of determining the induced mass of the entrained fluid.

CONCLUSION

It is concluded that an insight into the mechanism of induced mass will not be gained without a mathematical analysis of flow regimes.

REFERENCES

1. *Proposal for the Experimental Investigation of Induced Mass and Drag of the Basic Finner Missile*, A. V. Hershey, W. E. Moyer, D. P. Fields, NPG Tech Memo No. K-11/58 (dated July 1958)
2. Ltr from R. E. Prowse, (Davidson Laboratory) to F. D. Donoghue (Bureau of Ordnance) dated 3 Oct 1959
3. Ltr from P. W. Brown (Davidson Laboratory) to A. V. Hershey (Naval Weapons Laboratory) dated 21 Sept 1960
4. *Added-Mass and Drag Coefficients of Basic Finner Missile*, D. Savitsky and R. E. Prowse, Davidson Laboratory Report No. R-824 (dated December 1960)
5. *Exact Solution of the Neumann Problem. Calculation of Non-Circulatory Plane and Axially Symmetric Flows about or within Arbitrary Boundaries*, A. M. O. Smith and J. Pierce, Douglas Aircraft Company Report No. ES 26988 (dated 25 April 1958)

APPENDIX A

TABLES

TABLE I

CATALOG OF RECORDS FROM ORIGINAL DATA

Run Number	Sting Diameter (in)	Speed Range (ft)/(sec)	Scale Division (ft)/(sec)	Run Number*
2	1.75	0 - 11.28	0.12	1
6	1.75	0 - 10.45	0.12	2
9	1.75	0 - 17.22	0.12	3
20	1.75	6.12 - 11.14	0.12	5
21	1.75	11.76 - 16.50	1.8	6
22	1.75	9.85 - 5.86	1.8	7
25	1.75	14.06 - 8.40	1.8	9
27	1.75	18.18 - 14.00	1.8	10
28	1.75	9.70 - 6.50	1.8	8
33	2.50	0 - 11.96	1.8	11
33	2.50	11.96 - 5.72	1.8	13
39	2.50	13.63 - 17.50	1.8	12
47	2.50	17.69 - 14.00	1.8	14
54	1.75	0 - 16.22	1.8	4

*Runs as renumbered in the final report, reference 4.

TABLE II

DRAG COEFFICIENTS FROM CONSTANT SPEED RUNS AND AFTER ADJUSTMENT FOR OPTIMUM ERROR CURVES

j	v_j	k_j Constant Speed	k_j Optimum Error
0	0	0.0372	0.0372
2	1	0.0372	0.0372
4	2	0.0372	0.0372
6	3	0.0372	0.0372
8	4	0.0363	0.0363
10	5	0.0354	0.0354
12	6	0.0346	0.0346
14	7	0.0341	0.0341
16	8	0.0381	0.0381
18	9	0.0392	0.0392
20	10	0.0375	0.0354
22	11	0.0354	0.0353
24	12	0.0352	0.0352
26	13	0.0347	0.0347
28	14	0.0344	0.0344
30	15	0.0341	0.0344
32	16	0.0340	0.0344
34	17	0.0339	0.0344
36	18	0.0339	0.0344
38	19	0.0338	0.0344
40	20	0.0338	0.0344

APPENDIX B

SAMPLE RECORDS

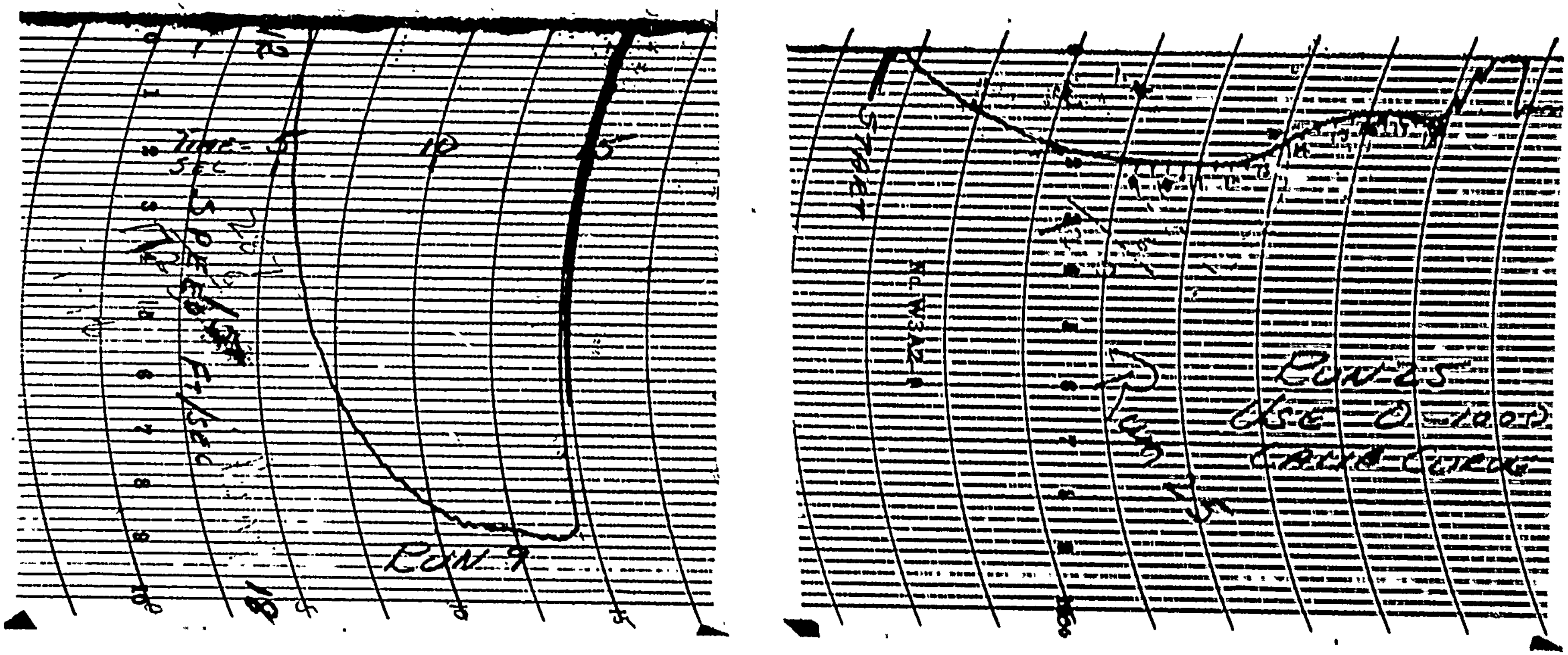

Figure 1 - Sample Speed Records

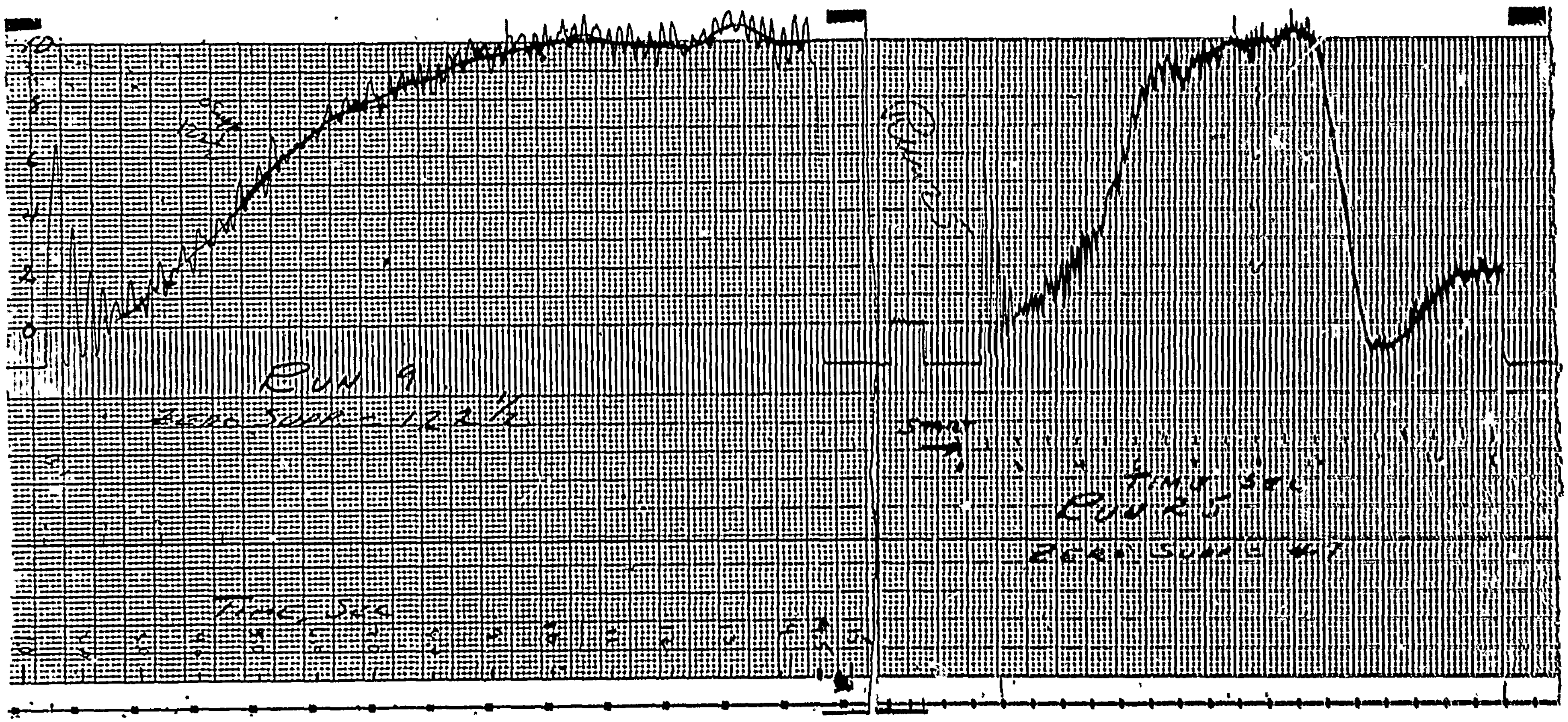

Figure 2 - Sample Drag Records

APPENDIX C

Blocks of Input Data for Trajectory Integrations.

Block 0000. Values of m_j (slugs) for $0 \leq j \leq 40$.
NORC numbers with v_j in the range 0 (0.5) 20.

Block 0001. Values of k_j (slugs)/(ft) for $0 \leq j \leq 40$.
NORC numbers with v_j in the range 0 (0.5) 20.

First Block with Block Number equal to Run Number.
Special numbers: $\underbrace{\text{xxxx.xxxx}}_{t_i\ \text{(sec)}}$ $\underbrace{\text{xxxx.xxxx}}_{v_i\ \text{(ft)/(sec)}}$

Second Block with Block Number equal to Run Number.
Special numbers: $\underbrace{\text{xxxx.xxxx}}_{\bar{v}_i^2\ \text{(ft)}^2/\text{(sec)}^2}$ $\underbrace{\text{xxxx.xxxx}}_{\bar{f}_i\ \text{(lb)}}$

The number of intervals determines the length of block.

PAGE 1

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PAGE 2

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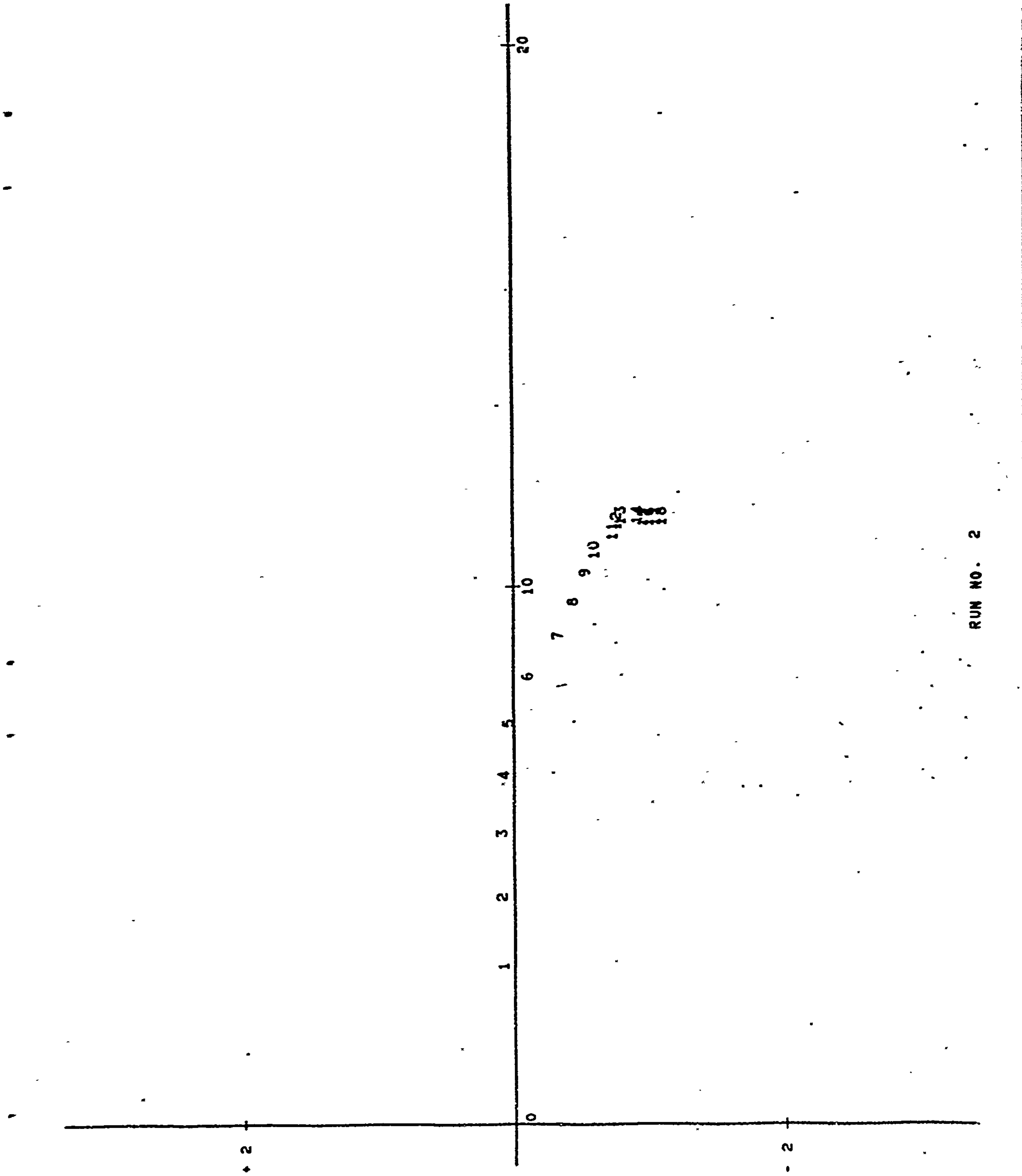
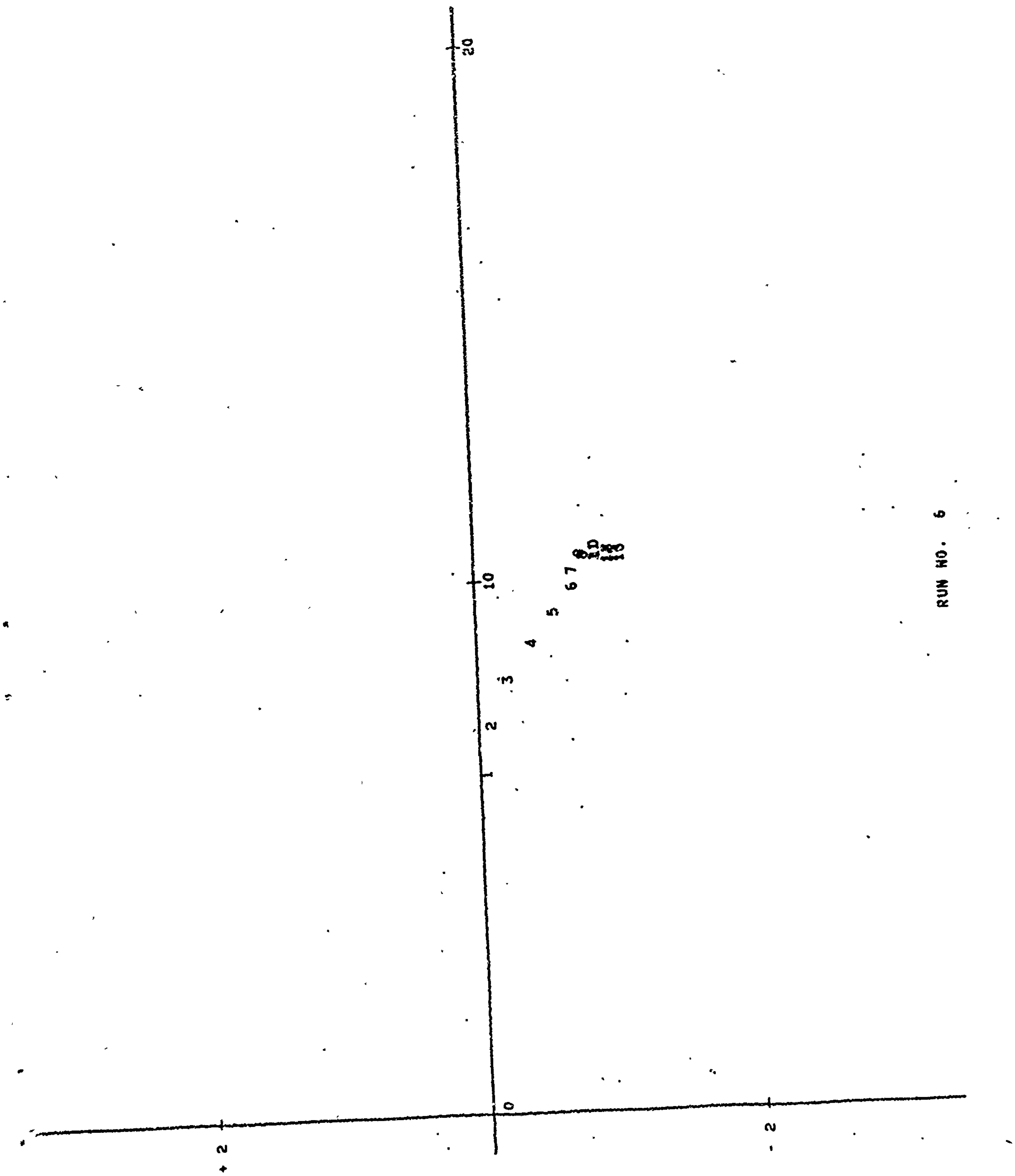
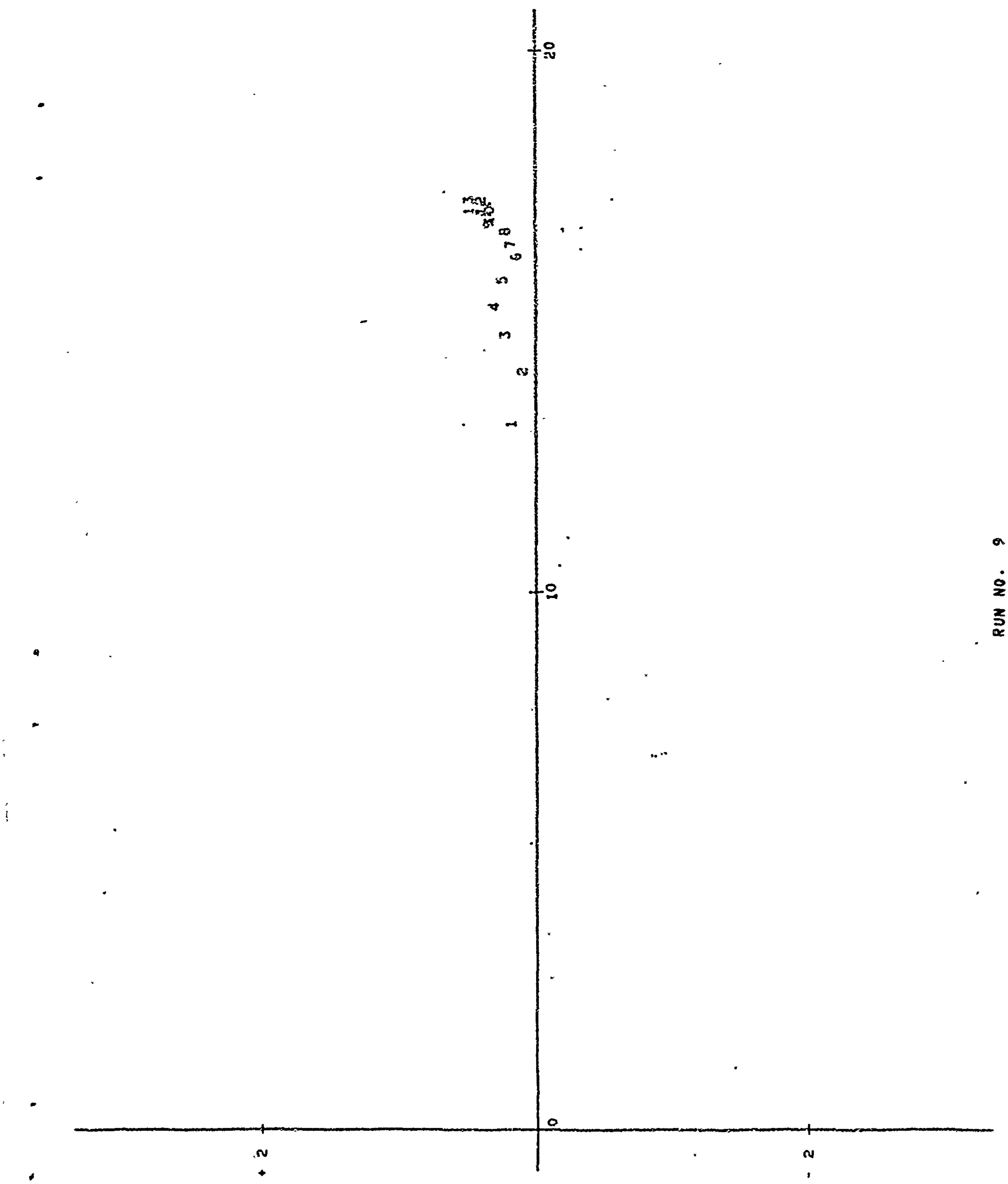
0027	0001	BOB	12 94 1201 1210 0027	00 00 0000 0000 0000	03 22 2000 0012 2400	03 27 2600 0012 5100	
	0002		03 25 4600 0011 9600	03 12 4600 0010 1100	02 85 9100 0008 3900	02 55 3400 0007 6300	
	0003		02 37 2000 0007 7300	02 26 5200 0007 8800	02 19 0500 0007 7300	12 94 1201 1210 0027	EOB
0028	0001	BOB	12 94 1101 1110 0028	00 05 0000 0009 7000	00 05 5000 0009 7000	00 06 0000 0009 7000	
	0002		00 06 5000 0009 7000	00 07 0000 0009 7000	00 07 5000 0009 6000	00 08 0000 0009 1000	
	0003		00 08 5000 0008 4000	00 09 0000 0007 5000	00 09 5000 0006 5000	12 94 1101 1110 0028	EOB
0028	0001	BOB	12 94 1201 1210 0028	00 00 0000 0000 0000	00 94 0900 0003 5500	00 94 0900 0003 5000	
	0002		00 94 0900 0003 5500	00 94 0900 0003 5000	00 93 1200 0003 3000	00 87 4800 0002 8500	
	0003		00 76 6800 0001 1000	00 63 4000 0000 5500	00 49 2500 1000 2000	12 94 1201 1210 0028	EOB
0054	0001	BOB	12 94 1101 1123 0054	00 01 0000 0002 6600	00 01 5000 0005 3000	00 02 0000 0007 0600	
	0002		00 02 5000 0008 4000	00 03 0000 0009 6000	00 03 5000 0010 7000	00 04 0000 0011 6400	
	0003		00 04 5000 0012 6500	00 05 0000 0013 4700	00 05 5000 0013 9000	00 06 0000 0014 4000	
	0004		00 06 5000 0014 7000	00 07 0000 0015 0000	00 07 5000 0015 2500	00 08 0000 0015 5000	
	0005		00 08 5000 0015 6500	00 09 0000 0015 7700	00 09 5000 0015 9500	00 10 0000 0016 0500	
	0006		00 10 5000 0016 2200	00 11 0000 0016 2200	00 11 5000 0016 2200	00 12 0000 0016 2200	
	0007		12 94 1101 1123 0054	EOB			
0054	0001	BOB	12 94 1201 1223 0054	00 00 0000 0000 0000	00 17 5800 0004 8000	00 38 9700 0004 9500	
	0002		00 60 2000 0005 2200	00 81 3600 0005 5850	01 03 3300 0006 1500	01 24 9900 0006 5600	
	0003		01 47 7600 0007 1800	01 70 7300 0007 6000	01 87 3300 0007 8600	02 00 2800 0008 3400	
	0004		02 11 7200 0008 3900	02 20 5400 0008 7100	02 28 7800 0008 8100	02 36 4100 0008 9400	
	0005		02 42 5900 0009 0400	02 46 8100 0009 0400	02 51 5500 0009 2500	02 56 0000 0009 5100	
	0006		02 60 3500 0009 3000	02 63 0900 0009 4000	02 63 0900 0009 2500	02 63 0900 0009 4500	
	0007		12 94 1201 1223 0054	EOB			
			00 00 0000 0000 0000	EOF			

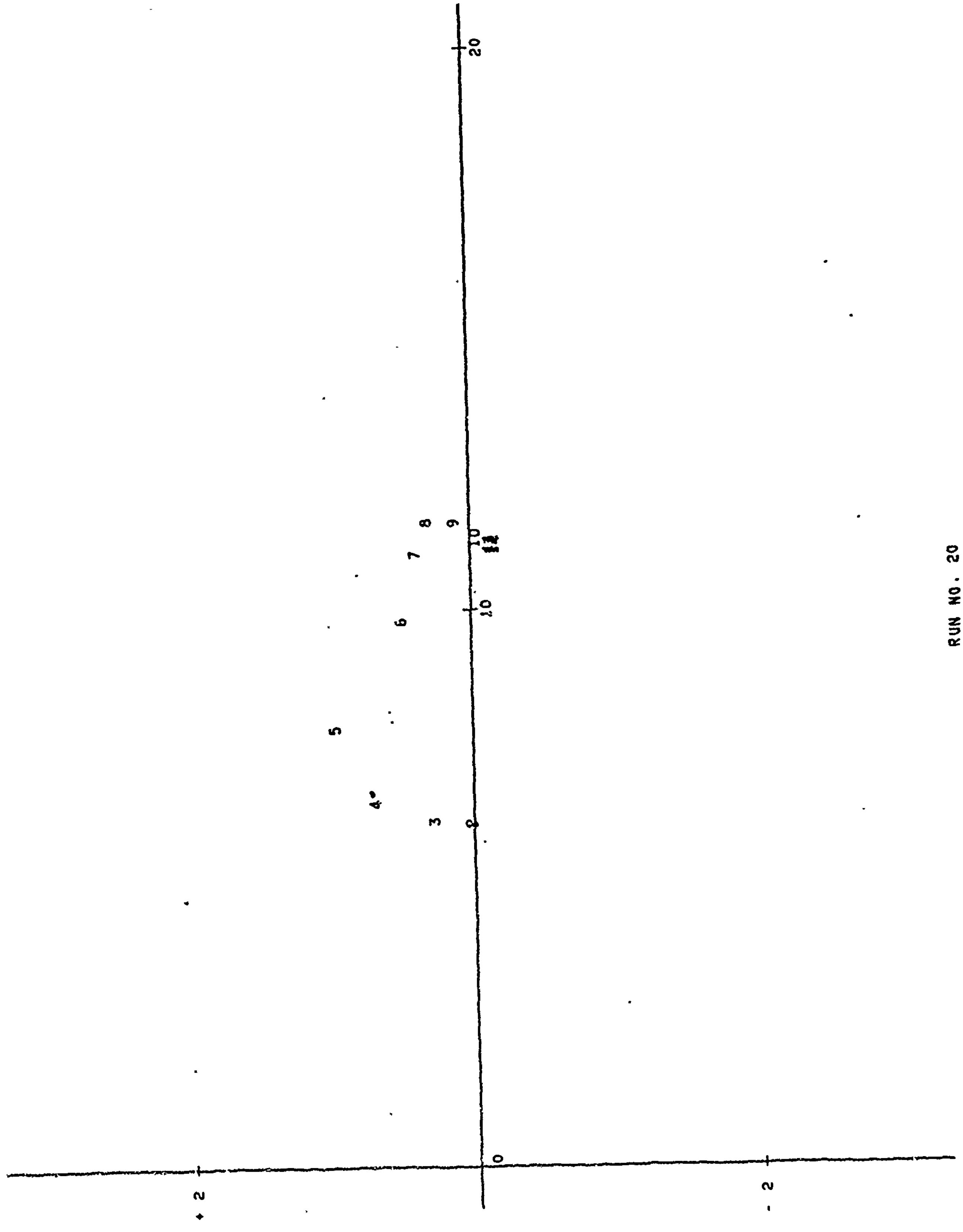
APPENDIX D

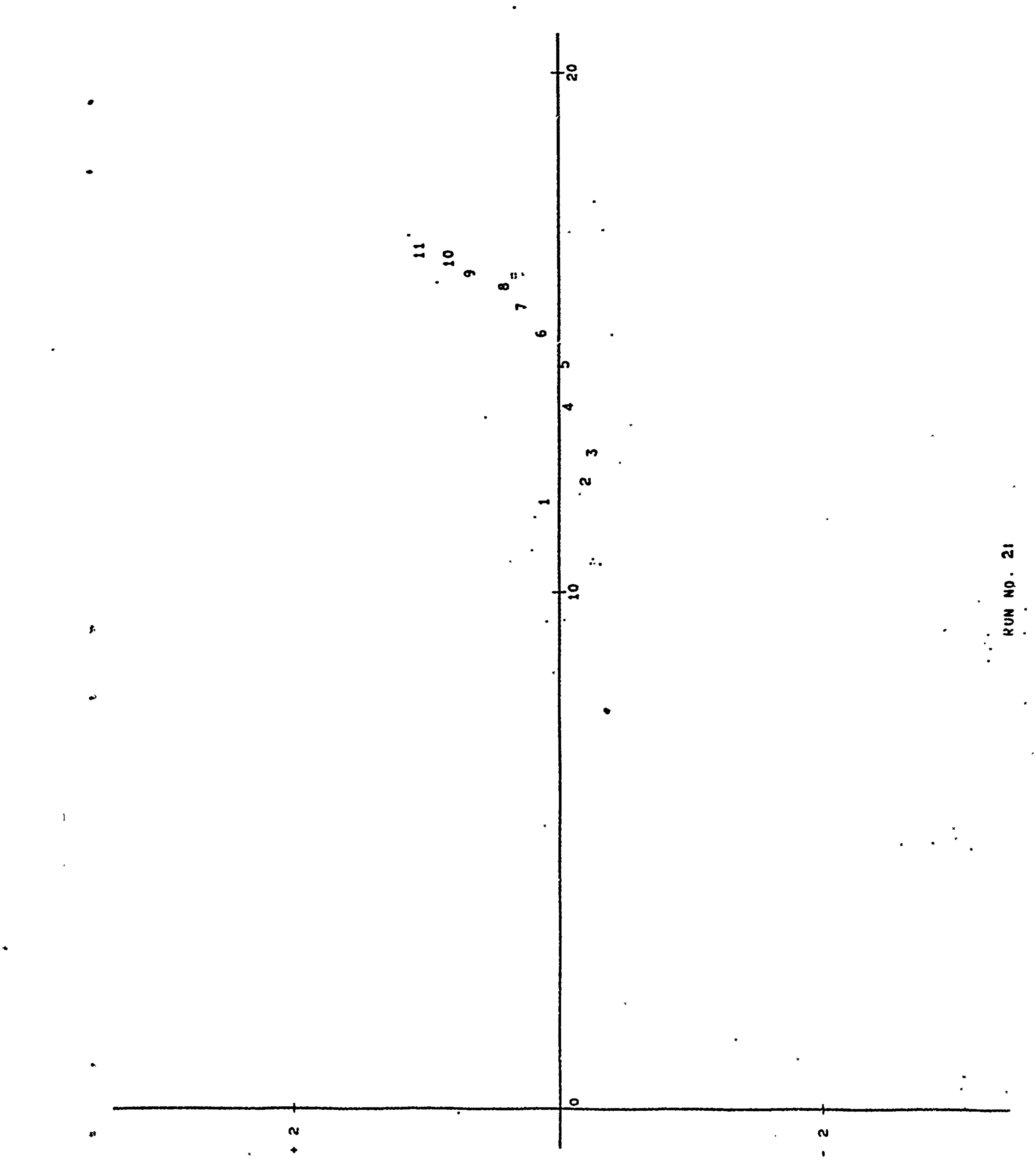
Plots of ε_n (ordinates) versus v_n (abscissae) for each run with each point marked by the value of n. (Sting diameter = 1.75 in. The data for the larger sting diameter ran off scale.)

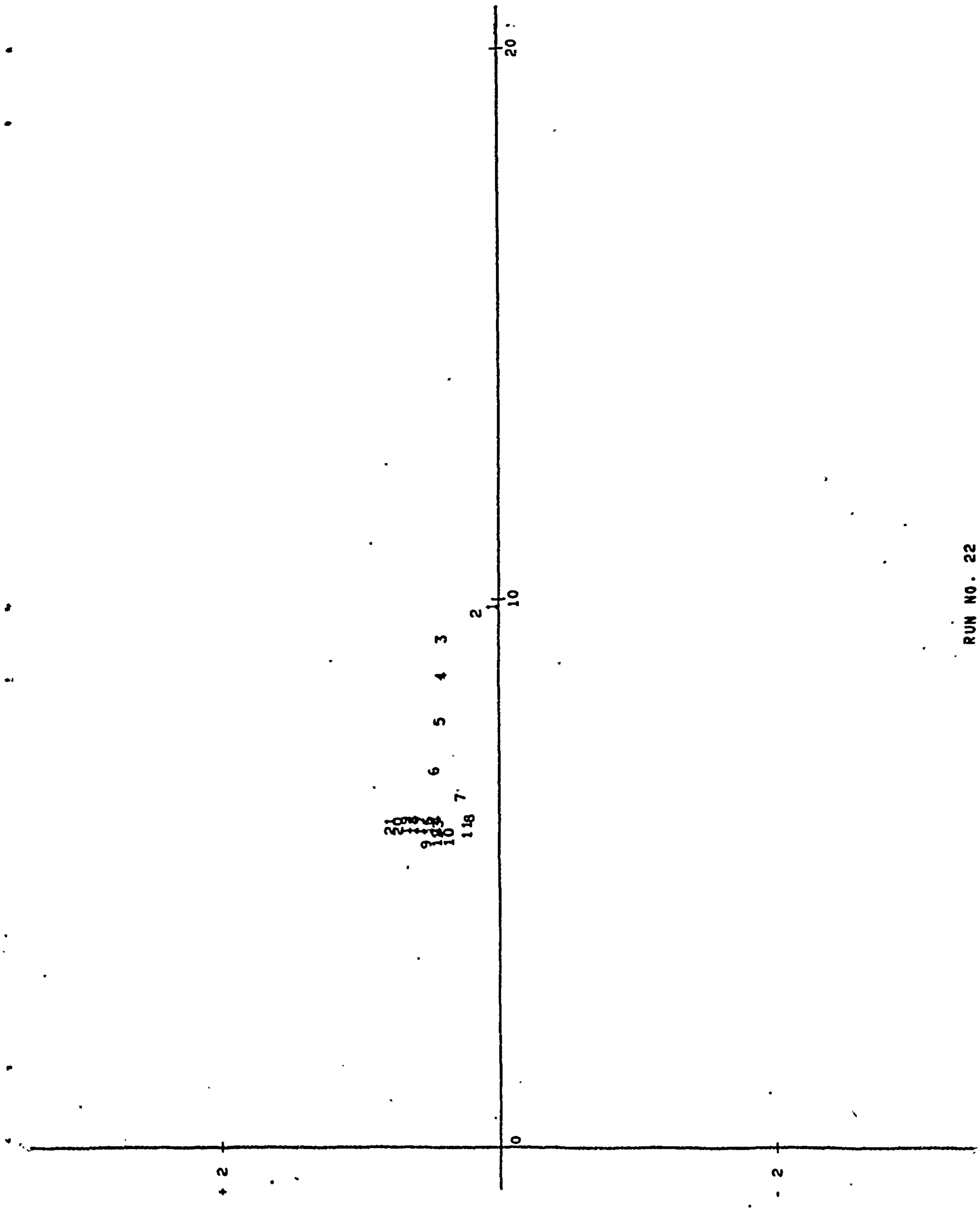

RUN NO. 2


20
10
2
1
0
+ 2
- 2
RUN NO. 6

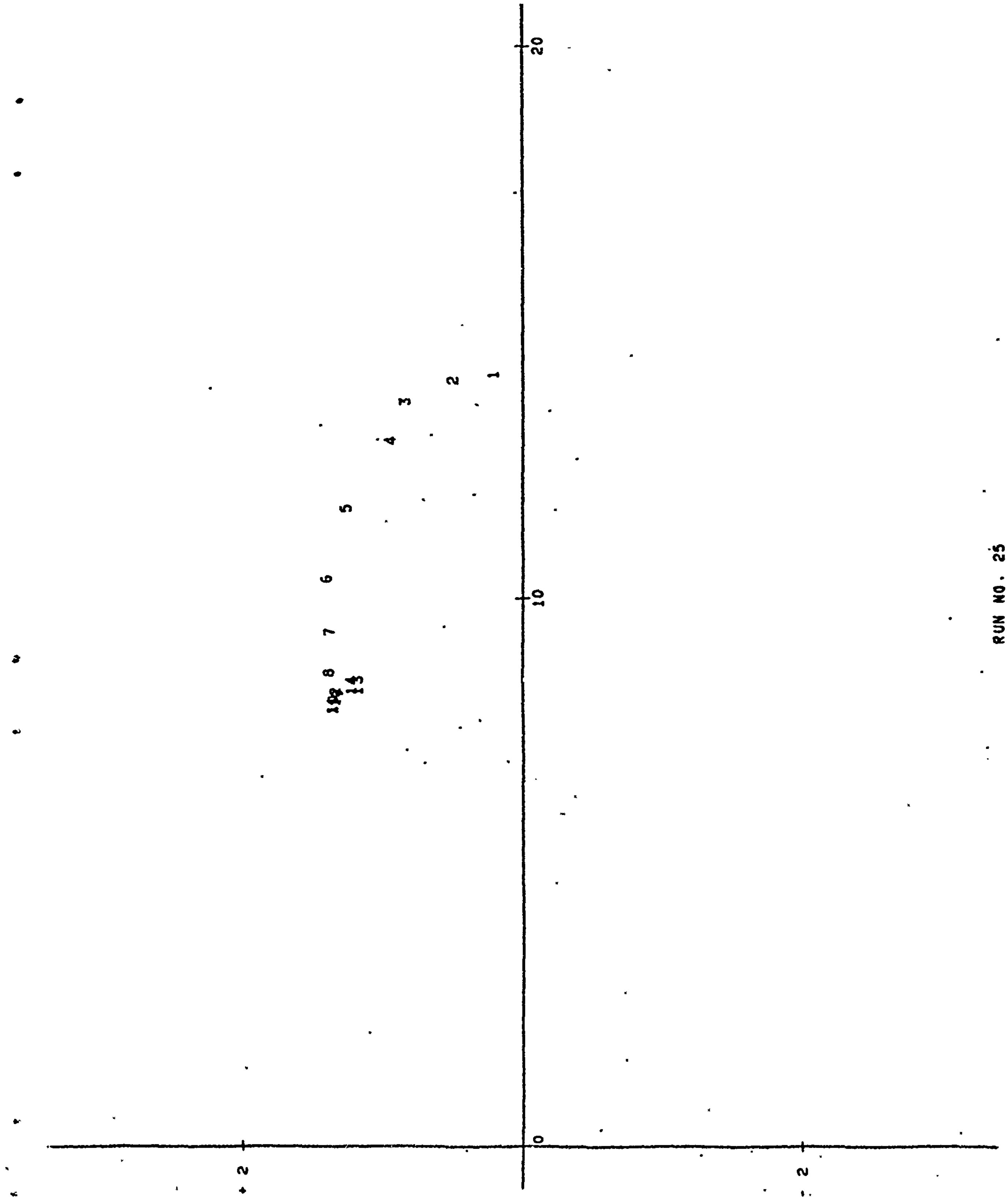

RUN NO. 9
20
10
0
+.2
-.2

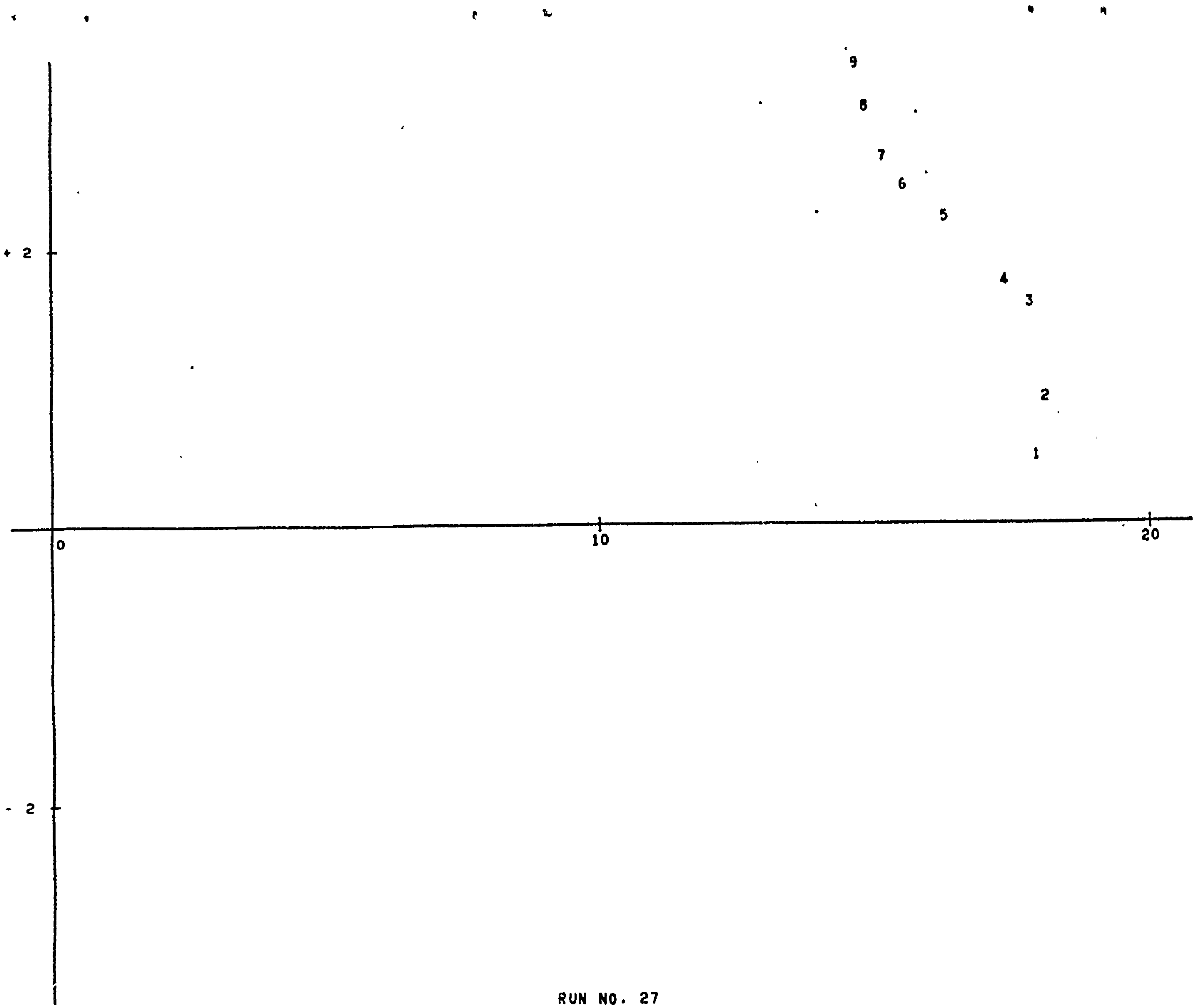

RUN NO. 20
20
10
0
+ 2
- 2


RUN NO. 21
20
10
5
0
+2
-2
1
2
3
4
6
7
8
9
10
11

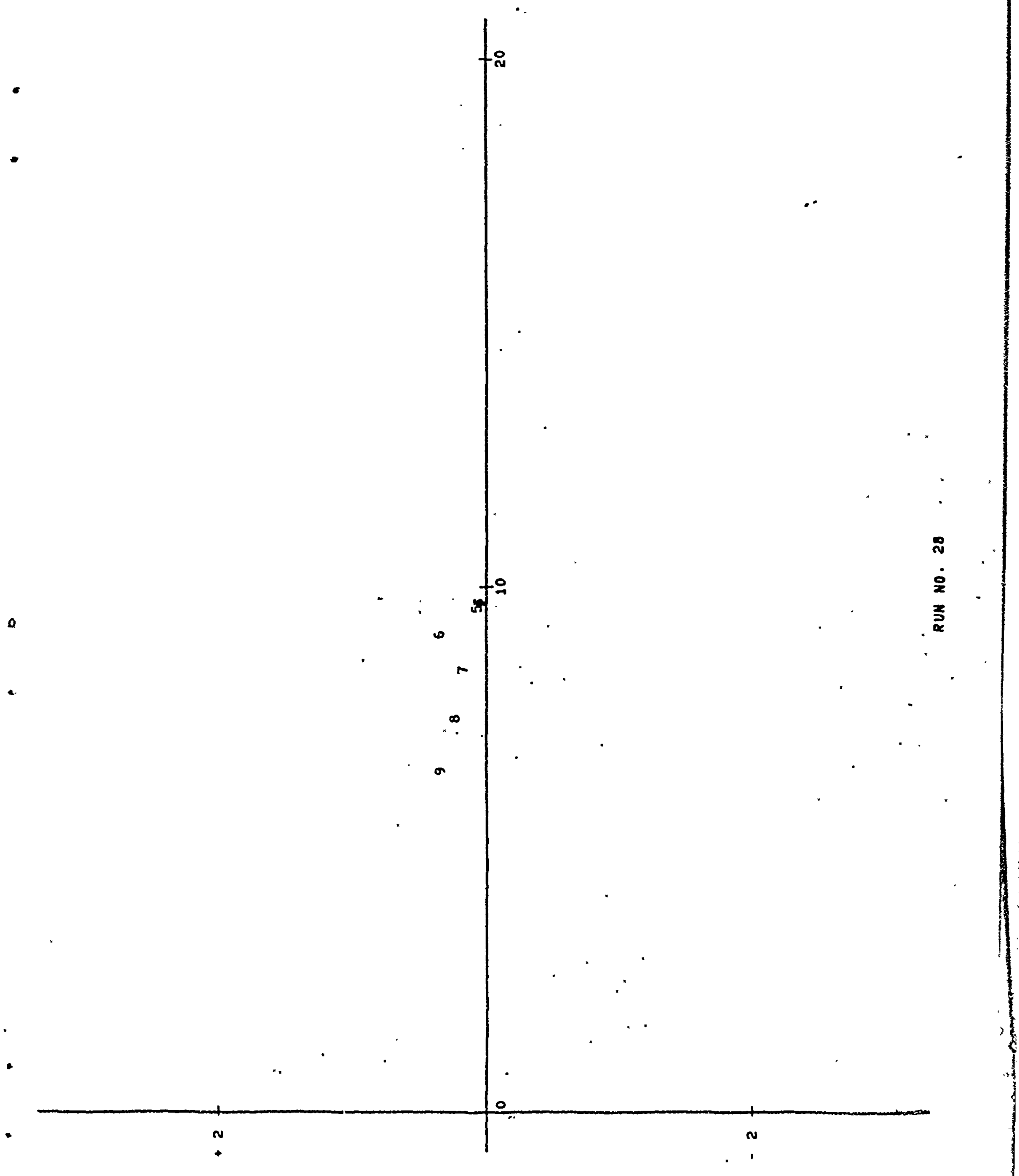


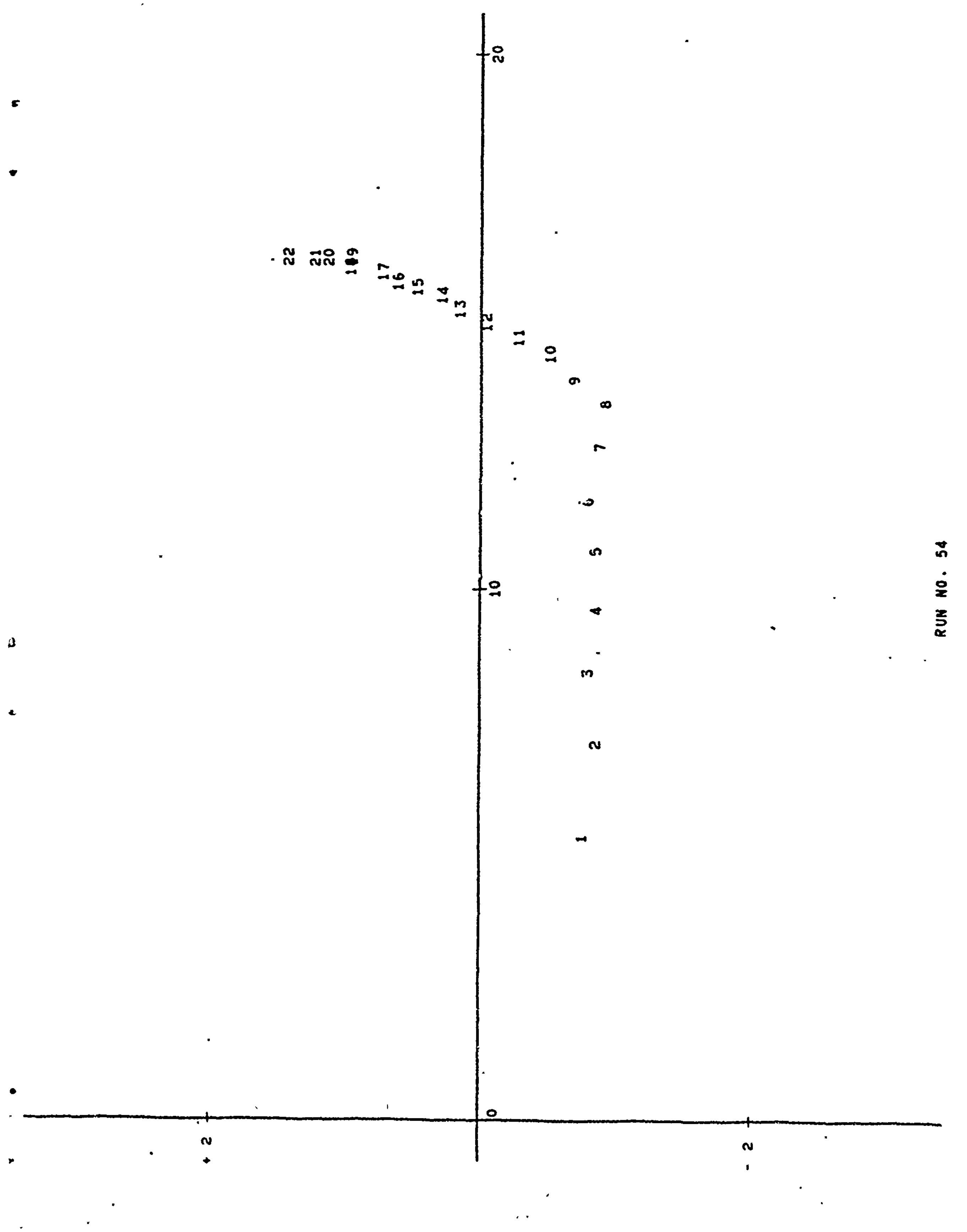
RUN NO. 22
20
10
0
2
-2


RUN NO. 25



RUN NO. 27


RUN NO. 28
20
10
0
2
-2
5
6
7
8
9


RUN NO. 54